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WILL THE GREEN TRANSITION BE INFLATIONARY? EXPECTATIONS MATTER

by Alessandro Ferrari* and Valerio Nispi Landi*

Abstract

We analyse a progressive increase in the tax on emissions in a simple two-period New Keynesian model with an AS-AD representation. We find that the increase in the tax today exerts inflationary pressures, but the expected further increase in the tax tomorrow depresses current demand, putting downward pressure on prices: we show that the second effect is larger. However, if households do not anticipate a future fall in income (because they are not rational or the government is not credible), the overall effect of the transition may be inflationary in the first period. We extend the analysis in a medium-scale DSGE model and we find again that the green transition is deflationary. Also in this larger model, by relaxing the rational expectations assumption, we show the transition may initially be inflationary.

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1 Introduction[1]

The EU aims to be climate-neutral by 2050, i.e. having an economy with net-zero greenhouse gas emissions. This in line with the obligations of the Paris Agreement, which has the goal of limiting global warming to well below 2°C and pursuing efforts to limit it to 1.5°C.

There are several studies analyzing the impact of the green transition on macroeconomic variables.[2] The common denominator of this literature is that the green transition will be costly in terms of output: the introduction of an emission tax induces firms to reduce production and pay abatement costs. Instead, the response of inflation along the green transition has been less explored. This issue is of utmost importance for central banks, as highlighted by the debate on the nature and persistence of the recent surge in inflation arising from high energy prices, supply-bottlenecks, and pent-up demand. On January 8th 2022, in a speech given at the American Finance Association, the ECB Executive Board member Isabel Schnabel warned that the green transition poses upside risks to medium-term inflation.

In this paper, we analyze the impact of the green transition on inflation: first, we use a simple two-period New Keynesian model to derive analytical results; second, we use a larger framework for a more quantitative simulation. We model the green transition as a gradually increasing emission tax. We show that this environmental policy has two opposite effects on prices. On the one hand, emission taxes operate as a negative supply shock as they raise firms' marginal costs, creating inflationary pressures; on the other hand, emission taxes act as negative demand shocks, as the reduction in expected future labor income and profits induces rational agents to cut current consumption and investment, creating deflationary pressures. Under the assumption of perfect foresight, the second effect dominates and the green transition is deflationary in the short- medium-term, consistently with the empirical estimates of Konradt and Weder (2021). If we assume that the future emission taxes are not fully credible, so that agents do not factor in the future income fall, the transition is inflationary in the early stage.[3]

In the rest of the paper, we set up a small-scale model (Section 2), a medium-scale model (Section 3), and we draw some concluding remarks (Section 4).

[1]We thank Pietro Catte, Paolo Del Giovane, Stefano Neri, Francesco Paternò, and Alessandro Secchi for useful feedback. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Italy.

[2]Some recent examples are Carattini et al. (2021), Diluiso et al. (2021), Ferrari and Pagliari (2021), and Ferrari and Nispi Landi (2022).

[3]The assumption of imperfect credibility is in line with the literature that tries to solve the "Forward guidance puzzle". Indeed, imperfect credibility of the public sector makes current output less dependent on future policy variables.

2 Green transition and inflation

2.1 Assumptions

Following Benigno (2015), we set up a two-period New Keynesian model. We interpret the first period as the short run and the second period as the long run. In the short run, only some firms can change prices; the others keep them at a predetermined level, which was set prior to the realization of the short-run shock. This assumption implies an upward-sloping Phillips curve while, in the long run, all firms optimize and the Phillips curve is vertical. In this framework monetary policy has real effects only in the short run. Labor is the only factor of production. Following the environmental literature, we assume that production generates CO2 emissions, that can be dampened by a costly abatement technology. For simplicity, we assume that emissions affect neither utility nor production. Households supply labor to firms, choose consumption, and the amount of a nominal risk-free bond. The government sets an emission tax, transferring the revenues to households in lump-sum fashion; the central bank sets the interest rate in the short run and determines the price level in the long run. We denote long-run variables with a bar. We leave the detailed description of the model in Appendix A.

After a first-order Taylor approximation, the model can be described by two equations:

$$\begin{aligned} y &= \bar{y} - \frac{1}{\sigma}\left[i - (\bar{p} - p) - \rho\right] && (1) \\ p &= p^e + \kappa\left(y - y^n\right), && (2) \end{aligned}$$

where y is output and p denotes the price level, both expressed in logs; i is the nominal interest rate; p^e is the price set by those firms that cannot change their price: p^e can be interpreted as the previous-period price level, assuming that in the previous period the model was in the steady state; y^n is the natural level of output, i.e. the production level prevailing if prices were fully flexible, given by (up to an uninteresting constant):

$$y^n = -\frac{\mu}{\sigma + \eta}\tau, \tag{3}$$

where τ is the emission tax; given that in the long run prices are fully flexible, long-term output $\bar{y}$ is equal to the long-run natural level:[4]

$$\bar{y} = -\frac{\mu}{\sigma + \eta}\bar{\tau}. \tag{4}$$

[4]In equilibrium, short-run emissions e are given by $e = y - \frac{\tau}{\nu}$; the same equation holds in the long run.

Parameters σ, η, κ, ρ, and μ are all positive.[5]

Equation (1) is a standard Euler equation, that can be interpreted as an aggregate demand (AD) curve with slope $-\sigma$ in the space $\{y, p\}$: for a given monetary policy $\{i, \bar{p}\}$, a higher price level today implies lower inflation expectations, resulting in a higher real interest rate that depresses demand. Equation (2) is a Phillips curve, that can be interpreted as an aggregate supply (AS) curve with slope κ in the space $\{y, p\}$.

2.2 Monetary and fiscal policies

We assume that the central bank sets the interest rate using a Taylor rule:

$$i = \rho + \phi \left(p - p^e\right), \tag{5}$$

where $\phi > 1$, which means that the central bank raises the interest rate if the current price level p is above the predetermined level p^e, which can be interpreted as the previous-period price level: this rule is equivalent to an inflation targeting in the two-period model we consider. This monetary rule flattens the AD curve, whose slope becomes $-\frac{\sigma}{1+\phi}$: when $\phi \to \infty$, the AD curve is flat, as the central bank does not allow for any price fluctuations.

We also assume that the long-run price level $\bar{p}$ is set exogenously by a perfectly-credible central bank.[6] The government sets short- and long-run emission taxes $\{\tau, \bar{\tau}\}$; as in Ferrari and Nispi Landi (2022), we assume that the green transition is driven by a gradually increasing emission tax:

$$\bar{\tau} > \tau > 0. \tag{6}$$

This assumption captures the European goal to become gradually climate-neutral by 2050. In our baseline exercise, we assume that there is no uncertainty: agents perfectly foresee long-term variables. In the following sections we prove that this is a key assumption for the effects of the green transition on the price level and we also show how results change under imperfect foresight.

2.3 Analysis

We depict the short-run initial equilibrium in Figure 1. In the short run, before the government announces the environmental plan, output is at its natural level, prices are equal to the predetermined level (equal to p^e in Figure 1): the economy is in point E. The

[5] σ is the coefficient of relative risk aversion; η is the inverse of the Frisch elasticity; $\kappa \equiv \frac{(\sigma+\eta)(1-\lambda)}{\lambda}$, where λ is the share of firms that cannot adjust their prices; ρ is the subjective discount rate; $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$, where ε is the elasticity of substitution between goods produced by different firms.

[6] By adding a money demand function to the model, we could assume that the central bank sets the long-run money supply to obtain the desired long-term price level.

long-run equilibrium is given by a vertical Phillips curve (i.e. the definition of long-term natural output) and by the long-term price level chosen by the central bank.

The environmental plan implies a short-run increase in the emission tax. Other things equal, marginal costs rise and those firms that are allowed to optimize increase their price: the AS curve shifts upward (Figure 1 red dotted line). But other things are not equal. While the short-term emission tax does not shift the AD curve, a rise in the long-term tax reduces long-term output, moving the AD curve downward (Figure 1 blue dotted line): households foresee a lower future income and reduce current demand. Short-term output unambiguously falls, given the reduction in aggregate demand and aggregate supply (point E'). The short-term impact on prices is less obvious: while the increase in marginal costs yields positive supply-side price pressures, the decrease in future income yields negative demand-side price pressures. Solving the system, we find that the demand effect prevails:

$$p = \Phi - \frac{\sigma\mu\kappa}{\left[\sigma + \kappa\left(1+\phi\right)\right]\left(\sigma+\eta\right)}\left(\bar{\tau} - \tau\right), \tag{7}$$

where $\Phi > 0$. Given $\bar{\tau} > \tau > 0$, an assumption consistent with global plans to progressively tighten the emissions regulation, the introduction of a carbon tax necessarily exerts a negative impact on prices. In the new equilibrium, E', the new short-run natural level $y^{n\prime}$ is lower than the natural level y^n before the tax announcement; the new short-run output y' is below its natural level, inducing a fall in prices.

Green transition

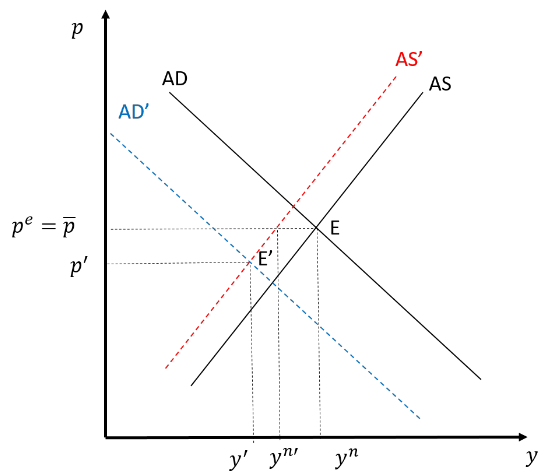

Figure 1: The short-equilibrium.

The short-term fall in prices hinges on a crucial assumption: households perfectly foresee long-term income $\bar{y}$, which means that they foresee the long-term environmental tax $\bar{\tau}$. Perfect foresight means that households immediately anticipate the future fall in income arising from the green transition and reduce current consumption: this mechanism shifts the AD downward, decreasing the price level. There are several reasons why economic agents may fail in correctly anticipate the future tax. For instance, households may have sticky information à *la* Mankiw and Reis (2002) because macroeconomic news spread slowly through the population. Or households may not believe ex-ante to the policy announcement of future fiscal plans, as in Lemoine and Lindé (2016).

Denote with an f the expectation on long-term variables. Under perfect foresight $\bar{\tau}^f = \bar{\tau}$; suppose instead that a fraction $1-\delta$ of households believes that the emission tax increase is transitory and in the long term $\bar{\tau} = 0$:[7]

$$\bar{\tau}^f = \delta\bar{\tau}. \tag{8}$$

In this case, the price level reads:

$$p = \Phi - \frac{\sigma\mu\kappa}{\left[\sigma + \kappa\left(1+\phi\right)\right]\left(\sigma+\eta\right)}\left(\delta\bar{\tau} - \tau\right), \tag{9}$$

which means that if the share of households with perfect foresight is low enough ($\delta < \frac{\tau}{\bar{\tau}}$), the green transition is inflationary, as the downward shift in the AD curve does not offset the upward shift in the AS curve. Expectations of households on the future carbon tax are the key factor that determines the response of inflation after the announcement of a new environmental plan to curb emissions.

3 Extensions

Our two-period model has been kept purposely simple to derive closed-form results, thus we want to verify that our findings hold also in a larger and more realistic model. We assume that households are infinitely lived, capital is a further factor of production, and that firms pay price adjustment costs. As in Ferrari and Nispi Landi (2022), we assume that the government announces the introduction of an emission tax that increases linearly for 30 years, reaching a level high enough to induce firms to abate all the emissions. In order to fully abate emissions, our calibration implies that the price of one ton of CO2 should be around 65 euro. We calibrate the model on euro-area data. We leave the detailed description of the model in Appendix B.

Figure 2 shows the first 10 years of the transition to the new steady state with zero emissions.[8] In order to capture imperfect credibility of the government, we assume that households and firms initially believe that the current tax rate is transitory and follows an $AR(1)$ process with parameter 0.5, and that they start believing the announced persistent

[7] We keep assuming that the foresight on the future price level is correct.

[8] We plot only the initial phase of the transition to better highlight the initial response of inflation.

environmental plan only after F quarters of its actual implementation. We consider a baseline scenario where economic agents immediately believe the government (Figure 2 blue solid line) and three alternative scenarios with $F = \{5, 10, 15\}$ (Figure 2 black dashed, red dotted, and green solid line, respectively).

When households immediately believe the new environmental plan, they anticipate the future fall in output and income, and therefore cut current consumption and investment. Prices fall immediately, despite the fact that the tax increases nominal marginal costs: the downward shift in aggregate demand is larger than the upward shift in aggregate supply, as in Figure 1.

If economic agents do not believe (or are poorly informed of) the environmental plan, the short-term adjustment in aggregate demand does not occur (Figure 2, black dashed, red dotted, and green solid line). Inflation increases due to the higher marginal costs until households and firms realize that the announced plan is actually true and then immediately adjust consumption, investment, and prices.[9]

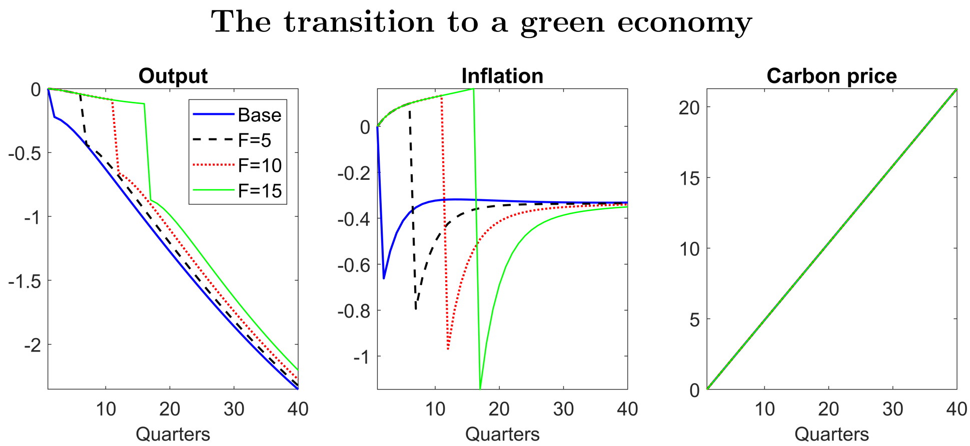

Figure 2: Transition to a zero-emission economy, driven by an emission tax. Output is in percentage deviations with respect to the value they would have had with no increase in the emission tax; inflation is in deviations compared to the target reported at annual rates; the price of carbon is in level deviations. The path for the emission tax is announced in period 0. Blue solid line: baseline scenario; black dashed line: $F = 5$; red dotted line: $F = 10$; green solid line:$F = 15$.

4 Concluding remarks

A standard New Keynesian model suggests that the green transition is deflationary in the short and in the medium term. The key element behind this result is the expected fall in future income that depresses current aggregate demand, following the announcement of the environmental plan. Using a medium-scale DSGE model, we have also analysed

[9]The economy reaches a new steady state at the end of the transition, where output is lower, and inflation comes back to the initial level.

one potential scenario in which households do not initially expect a fall in future income, i.e. if they do not have perfect foresight, as result of sticky information or of the government policy's imperfect credibility. In this case, prices initially increase, until households become fully aware of the environmental policy plan.

There are other potential channels that we have not considered. For instance, another possible reason why households may not foresee a fall in future income would be if the green transition avoids a large fall in total factor productivity by preventing a large increase in global temperatures: if the expansionary effect of higher future productivity offsets the recessionary effect of higher future emission taxes, expected future income rises. In this case aggregate demand increases, and so does inflation. Moreover, the green transition could also raise aggregate output by triggering investment in green R&D that shift the technological frontier. We leave these scenarios to future research.

Appendix

A The 2-period model

The model consists of two periods: short and long term. In the short term, a fraction $1-\lambda$ of firms can change prices; a fraction λ keeps the price at its pre-determined level P^e. Variables in the long term are denoted with a bar.

A.1 Firms

A.1.1 Final-good firm

The representative final-good firm operates in perfect competition, producing the final good Y using a continuum of intermediate goods $Y(j)$:

$$Y = \left(\int_0^1 Y(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The following demand curve results for each input j:

$$Y(j) = \left(\frac{P(j)}{P}\right)^{-\varepsilon} Y,$$

where $P(j)$ is the price of input j and P is the price level:

$$P = \left(\int_0^1 P(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}.$$

A.1.2 Intermediate-good firms

A fraction $1-\lambda$ of firms choose the price in the short run. Their profit maximization problem reads:

$$\max_{P(j),Y(j),L(j),E(j),\gamma(j)} \Pi(j) = \frac{P(j)}{P}Y(j) - \frac{W}{P}L(j) - \tau E(j) - Z(j)$$

$$s.t. \begin{cases} Y(j) = L(j) \\ E(j) = (1-\gamma(j))\,Y(j) \\ Z(j) = \nu\left(\frac{\gamma(j)^2}{2}\right)Y(j) \\ Y(j) = \left(\frac{P(j)}{P}\right)^{-\varepsilon} Y, \end{cases}$$

where W denotes the nominal wage, L is the labor input, τ is an emission tax, E denotes emissions, Z denotes abatement spending, γ is the fraction of emissions abated. First order conditions with respect to abatement:

$$\tau - \nu\gamma(j) = 0$$
$$\gamma(j) = \gamma = \frac{\tau}{\nu}\ \forall j, \tag{A.1}$$

equal for every firm.[1] First order conditions with respect to $P(j)$ is

$$\frac{P(j)}{P} = \frac{\tilde{P}}{P} = \mu\left[\frac{W}{P} + \tau\left(1 - \frac{\tau}{2\nu}\right)\right], \tag{A.2}$$

where $\tilde{P}$ is the price set by each firm that can re-optimize the price in the short term. In the long run, all firms optimize, thus setting the same price:

$$\frac{\bar{P}(j)}{\bar{P}} = 1 = \mu\left[\frac{\bar{W}}{\bar{P}} + \bar{\tau}\left(1 - \frac{\bar{\tau}}{2\nu}\right)\right]. \tag{A.3}$$

In the long run abatement is given by:

$$\bar{\gamma} = \frac{\bar{\tau}}{\nu}. \tag{A.4}$$

A.2 Households

Households solve the following maximization problem:

$$\max_{C,L,\bar{C},\bar{L},B} \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\eta}}{1+\eta} + \beta\left[\frac{\overline{C}^{1-\sigma}}{1-\sigma} - \frac{\overline{L}^{1+\eta}}{1+\eta}\right]$$
$$s.t. \begin{cases} PC + B = WL + T \\ \bar{P}\bar{C} = \bar{W}\bar{L} + (1+i)B + \bar{T}, \end{cases}$$

where B denotes holding of a one-period bond, yielding a nominal rate i; T denotes lump-sum transfers from the government plus profits. The optimality conditions are the Euler equation:

$$C^{-\sigma} = \beta\overline{C}^{-\sigma}\frac{P(1+i)}{\overline{P}}, \tag{A.5}$$

[1]Solving the problem of the firms that cannot optimize prices, it is easy to see that they choose this fraction of abatement too.

and a labor supply condition in the short and in the long run:

$$C^{\sigma} L^{\eta} = \frac{W}{P} \tag{A.6}$$

$$\overline{C}^{\sigma} \overline{L}^{\eta} = \frac{\overline{W}}{\overline{P}}. \tag{A.7}$$

A.3 Market clearing

Given the aggregate labor supply

$$L = \int_0^1 L(j)\, dj,$$

the demand function of firm j in the short and in the long term are

$$L = YD \tag{A.8}$$

$$\bar{L} = \bar{Y}, \tag{A.9}$$

where:

$$D = \int_0^1 \left(\frac{P(j)}{P}\right)^{-\varepsilon} dj = P^{\varepsilon} \left[(1-\lambda)\, \tilde{P}^{-\varepsilon} + \lambda \left(P^{e}\right)^{-\varepsilon}\right] \tag{A.10}$$

is a measure of price dispersion, which is 1 in the long run. Aggregate emissions in the short term is

$$E = \int_0^1 E(j)\, dj = (1-\gamma)\, YD, \tag{A.11}$$

and in the long term

$$\bar{E} = (1-\bar{\gamma})\, \bar{Y}. \tag{A.12}$$

Aggregate abatement spending in the short term is

$$Z = \int_0^1 Z(j)\, dj$$

$$Z = \nu \left(\frac{\gamma^2}{2}\right) YD,$$

and in the long term

$$\bar{Z} = \nu \left(\frac{\bar{\gamma}^2}{2}\right) \bar{Y}.$$

Bonds are in zero net supply:

$$B = 0.$$

The good market clears:

$$Y = C + Z \tag{A.13}$$

$$\bar{Y} = \bar{C} + \bar{Z}. \tag{A.14}$$

Finally, the price in the short term is

$$P = \left[(1-\lambda) \tilde{P}^{1-\varepsilon} + \lambda P^{e(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \tag{A.15}$$

A.4 Log-linearization

We obtain a system of 7 equations and 7 variables $\left\{ \tilde{P}, P, D, Y, \bar{Y}, C, \bar{C} \right\}$ by combining equations (A.1)-(A.15):

$$\begin{aligned}
\frac{\tilde{P}}{P} =& \mu \left[C^{\sigma} (YD)^{\eta} + \tau \left(1 - \frac{\tau}{2\nu} \right) \right] \\
1 =& \mu \left[\bar{C}^{\sigma} \bar{Y}^{\eta} + \bar{\tau} \left(1 - \frac{\bar{\tau}}{2\nu} \right) \right] \\
C^{-\sigma} =& \beta \overline{C}^{-\sigma} \frac{P(1+i)}{\overline{P}} \\
Y =& C + \frac{\tau^2}{\nu 2} YD \\
\bar{Y} =& \bar{C} + \frac{\bar{\tau}^2}{\nu 2} \bar{Y} \\
D =& P^{\varepsilon} \left[(1-\lambda) \tilde{P}^{-\varepsilon} + \lambda (P^e)^{-\varepsilon} \right] \\
P =& \left[(1-\lambda) \tilde{P}^{1-\varepsilon} + \lambda P^{e(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}$$

The exogenous policy variables are $\left\{ \tau, \bar{\tau}, i, \bar{P} \right\}$. We log-linearize the previous 7 equations around an allocation without tax on emissions and flexible prices, denoting with a star such allocation. We also set $P^* = P^e$ and $i^* = \rho$. This implies:

$$\begin{aligned}
P =& \bar{P} = P^* \\
Y =& \bar{Y} = C = \bar{C} = Y^* \\
D^* =& 1.
\end{aligned}$$

By using the pricing condition in the long run we find that:

$$Y^* = \mu^{-\frac{1}{\sigma+\eta}}.$$

We use lower-case variables to denote the log of that variables, and the following approximation:

$$x - x^* = \log X - \log X^* \approx \frac{X - X^*}{X^*}.$$

Log-linearizing equation (A.10), we get that price dispersion is a second-order variable:

$$d = 0.$$

Log-linearizing equations (A.13) and (A.14), we get that abatement spending is 0 at a first order:

$$y = c$$

$$\bar{y} = \bar{c}.$$

Log-linearizing equation (A.15) we get:

$$p - p^* = (1 - \lambda)(\tilde{p} - p^*).$$

Log-linearizing the short-term pricing we get:

$$\tilde{p} - p = (\sigma + \eta)(y - y^n),$$

where y^n is the output level prevailing under flexible prices:

$$y^n = \frac{1}{\sigma + \eta} \log \mu^{-1} - \frac{\mu}{\sigma + \eta} \tau,$$

and a similar expression holds in the long run. Combining the previous conditions we get the AS curve:

$$p = p^e + \kappa (y - y^n),$$

where $\kappa \equiv \frac{(\sigma+\eta)(1-\lambda)}{\lambda}$. Log-linearizing the Euler equations, using the resource constraint, and $\log(1 + i) \approx i$, we get the AD curve:

$$y = \bar{y}^n - \frac{1}{\sigma}[i - (\bar{p} - p) - \rho],$$

where $\rho = -\log \beta$.

A.5 Solution

Assuming the central bank sets interest rates according to the Taylor rule,

$$i = \rho + \phi\left(p - p^{e}\right)$$

we can solve the AS-AD system for p as a function of $\{\tau, \bar{\tau}, \bar{p}\}$:

$$p = \Phi - \frac{\sigma\kappa\mu}{\left[\sigma + \kappa\left(1+\phi\right)\right]\left(\sigma+\eta\right)}\left(\bar{\tau} - \tau\right),$$

where $\Phi \equiv \frac{1}{\sigma+\kappa(1+\phi)}\left[\left(\sigma + \kappa\phi\right)p^{e} + \kappa\bar{p}\right]$.

B The Medium-Scale Model

In this section we describe the large-scale model used for the numerical simulation. The main different with the 2-period model is that households are infinitely lived and that capital is a further input of production. We also assume that labor-augmenting TFP z_t grows at a constant rate θ:

$$\frac{z_t}{z_{t-1}} = \theta.$$

Some variables of the model have a balanced growth path:

$$V_t = \frac{\tilde{V}_t}{z_t},$$

where $\tilde{V}_t$ is the detrended variable. We are going to express the model only in terms of detrended variables.

B.1 Households

The representative household solves the following optimization problem:

$$\max_{\{C_t, L_t, B_t, I_t, K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t}\left\{\log C_t - \frac{L_t^{1+\eta}}{1+\eta}\right\}$$

$$s.t. \begin{cases} C_t + \frac{B_t}{P_t} + I_t = \frac{r_{t-1}D_{Ht-1}}{P_t} + r_t^{k}K_{t-1} + w_t L_t - T_t + \Gamma_t \\ K_t = \left(1-\delta\right)K_{t-1} + \left[1 - \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - \theta\right)^{2}\right] I_t, \end{cases}$$

where C_t is consumption; L_t is labor; B_t is holding of public bonds, which yield a nominal gross rate r_t; I_t is investment in capital stock K_t, whose rental rate is r_t^k; $w_t \equiv \frac{W_t}{P_t}$ is the real wage, while W_t is the nominal wage and P_t is the CPI; T_t are lump-sum taxes; Γ_t denotes profits from firm's ownership. The first order conditions yield a labor supply expression:

$$\tilde{C}_t L_t^{\eta} = \tilde{w}_t. \tag{B.1}$$

The Euler equation for bonds:

$$1 = \beta \mathbb{E}_t \left(\frac{\tilde{C}_t}{\tilde{C}_{t+1}\theta} \frac{r_t}{\pi_{t+1}} \right), \tag{B.2}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate. The Euler equation for capital:

$$1 = \beta \mathbb{E}_t \left\{ \frac{\tilde{C}_t}{\tilde{C}_{t+1}\theta} \frac{\left[r_{t+1}^k + (1-\delta)\, q_{t+1}\right]}{q_t} \right\}, \tag{B.3}$$

where q_t is the lagrangian multiplier on the second constraint. The investment optimal condition:

$$1 = q_t \left\{ 1 - \frac{\kappa_I}{2} \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\theta - \theta \right)^2 - \kappa_I \frac{\tilde{I}_t}{\tilde{I}_{t-1}} \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\theta - \theta \right) \right\} + \kappa_I \beta \mathbb{E}_t \left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}\theta} q_{t+1} \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\theta \right)^2 \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\theta - \theta \right) \right]. \tag{B.4}$$

Finally, we write the law of motion of capital in terms of detrended variables:

$$\tilde{K}_t = (1-\delta) \frac{\tilde{K}_{t-1}}{\theta} + \left[1 - \frac{\kappa_I}{2} \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\theta - \theta \right)^2 \right] \tilde{I}_t. \tag{B.5}$$

B.2 Final-good firms

The representative final-good firm uses the following CES aggregator to produce Y_t:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $Y_t(j)$ is an intermediate input produced by the intermediate firm j, whose price is $P_t(j)$. The problem of the final-good firm is the following:

$$\max_{Y_t,\{Y_t(j)\}_{j\in[0,1]}} P_t Y_t - \int_0^1 P_t(j)\, Y_t(j)\, dj$$

$$s.t\ Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

This problem yields the following demand function $\forall j$:

$$Y_t(j) = Y_t \left(\frac{P(j)}{P_t}\right)^{-\varepsilon}.$$

B.3 Intermediate-good firms

B.3.1 Production and environment

There is a continuum of firms of measure one, indexed by i, producing a differentiated input through the following Cobb-Douglas function:

$$Y_t(j) = (K_{t-1}(j))^{\alpha} (z_t L_t(j))^{1-\alpha}.$$

Atmospheric carbon X_t is fueled by total domestic emission E_t:

$$E_t = \int_0^1 E_t(j)\, dj$$

and by exogenous rest-of-the-world emission $z_t E_t^{row}$:

$$X_t = (1-\delta^x) X_{t-1} + E_t + z_t E^{row},$$

where $1-\delta^x$ is the fraction of atmospheric carbon that remains in the atmosphere, and $z_t E^{row}$ are detrended rest-of-the-world emission, which are constant along a balanced growth path. Firm-level emissions are an increasing function of production:

$$E_t(j) = (1-\gamma_t(j))\, \nu_E Y_t(j),$$

where $\gamma_t(j)$ is the fraction of emission abated by firm j. Firm-level abatement costs Z_t are proportional to production and convex in the fraction of emissions abated:

$$Z_t(j) = Y_t(j) \frac{\nu_M}{1+\chi} \gamma_t(j)^{1+\chi}.$$

B.3.2 Firm's problem

Firms operate in monopolistic competition and pay quadratic adjustment costs $AC_t(j)$ in nominal terms, whenever they adjust prices with respect to the inflation target $\overline{\pi}$:

$$AC_t(i) = \frac{\kappa_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi}\right)^2 P_t Y_t.$$

Firms also pay a tax τ_t for each unit of emissions. The profit maximization problem of the generic firm j, expressed in terms of the domestic price index, is the following:

$$\max_{\{P_t(j),L_t(j),Y_t(j),K_{t-1}(j),E_t(j),\gamma_t(j)\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{\sum_{t=0}^{\infty} \beta^t \frac{c_t}{c_0}\left[\frac{P_t(j)}{P_t} Y_t(j) - w_t L_t(j) - r_t^k K_{t-1}(j) - \tau_t E_t(j) + \right.\right.$$

$$\left.\left. - Y_t(i)\frac{\nu}{1+\chi}\gamma_t(j)^{1+\chi} - \frac{\kappa_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi}\right)^2 Y_t\right]\right\}$$

$$s.t. \begin{cases} Y_t(i) = Y_t\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} \\ Y_t(j) = (K_{t-1}(j))^{\alpha}(z_t L_t(j))^{1-\alpha} \\ E_t(j) = (1-\gamma_t(j))\nu_E Y_t(j). \end{cases}$$

Firms choose same price, same inputs, and same output, so we can eliminate the index j. Optimal input demands:

$$r_t^k = mc_t \alpha\theta \frac{\tilde{Y}_t}{\tilde{K}_{t-1}} \tag{B.6}$$

$$\tilde{w}_t = mc_t(1-\alpha)\frac{\tilde{Y}_t}{L_t}, \tag{B.7}$$

where mc_t denotes real marginal costs gross of tax and abatement spending. We obtain the optimal abatement,

$$\gamma_t = \left(\frac{\nu_E}{\nu_M}\tau_t\right)^{\frac{1}{\chi}}, \tag{B.8}$$

and the optimal pricing equation,

$$\pi_t(\pi_t - \overline{\pi}) = \beta\mathbb{E}_t\left[\frac{c_t}{c_{t+1}}\frac{y_{t+1}}{y_t}\pi_{t+1}(\pi_{t+1} - \overline{\pi})^2\right] + \frac{\varepsilon}{\kappa_P}\left\{\left[mc_t + \nu_E\tau_t(1-\mu_t) + \frac{\nu_M}{1+\chi}\mu_t^{1+\chi}\right] - \frac{\varepsilon - 1}{\varepsilon}\right\}. \tag{B.9}$$

Finally, we rewrite the constraints and the law of motion of atmospheric carbon in terms of aggregate detrended variables:

$$\tilde{Y}_t = \left(\frac{\tilde{K}_{t-1}}{\theta}\right)^{\alpha} L_t^{1-\alpha}, \tag{B.10}$$

$$\tilde{E}_t = (1-\gamma_t)\,\nu_E \tilde{Y}_t, \tag{B.11}$$

$$\tilde{X}_t = (1-\delta^x)\,\frac{\tilde{X}_{t-1}}{\theta} + \tilde{E}_t + E^{row}. \tag{B.12}$$

B.4 Policy

We assume that the domestic bond market is frictionless and that lump-sum transfers are always adjusted to meet the desired level of government spending. Given these assumptions, the Ricardian equivalence holds, and the amount of outstanding public debt is not relevant. We set it at 0:

$$D_t = 0.$$

Public spending is financed with lump-sum and emission taxes:

$$G_t = T_t + \tau_t E_t,$$

and it is constant along a balance growth path:

$$\tilde{G} = \tilde{T}_t + \tau_t \tilde{E}_t.$$

We assume the following Taylor rule for r_t, including also an inertia component:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r} \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi(1-\rho_r)}. \tag{B.13}$$

B.5 Market Clearing

Clearing in the good market implies:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G} + \tilde{Y}_t \frac{\nu_M}{1+\chi}\gamma_t^{1+\chi} + \frac{\kappa_P}{2}\left(\pi_t - \overline{\pi}\right)^2 \tilde{Y}_t. \tag{B.14}$$

The equilibrium is described by a set of 14 equations (equation B.1-B.14) for the following 14 endogenous variables:

$$X_t \equiv \left\{\tilde{C}_t, \tilde{I}_t, \tilde{Y}_t, \tilde{K}_t, L_t, \tilde{w}_t, q_t, mc_t, \pi_t, r_t, r_t^k, \tilde{E}_t, \tilde{X}_t, \gamma_t\right\}.$$

The exogenous variable is τ_t. Moreover, we define p_t^C as the Euro price of one ton of CO2:

$$p_t^C = \frac{s_1 s_2}{s_3} \tau_t$$

where s_1

$$s_1 = \frac{Y^E}{\tilde{Y}}$$

and $Y^E = 3022.4$ is the steady-state quarterly euro-area GDP in EUR billions; we define s_2 as:

$$s_2 = \frac{X^{GtC}}{\tilde{X}}.$$

where $X^{GtC} = 870.1476$ is the stock of atmospheric carbon in 2019 in terms of GtC; $s_3 = 3.67$ is the number of CO2 units for 1 unit of carbon.

B.6 Calibration and solution

We calibrate the model to the euro area, at the quarterly frequency. We calibrate most economic parameters following the new version of the New Area-Wide Model (NAWM-II) in Coenen et al. (2018) (Table B.1). In the initial steady state we set $\mu = 0$. In the final steady state we set $\mu = 1$. As in Ferrari and Nispi Landi (2022), we assume that period 0 corresponds to 2019Q4, when the government introduces an emission tax that increases linearly for 120 quarters, such that from 2050 on all emissions are abated; in order to fully abate emissions, the carbon price is around 65 Euro per ton of CO2. In the baseline simulation we assume perfect foresight. In the other simulations we assume that households and firms believe for F periods that the current tax rate is transitory and follows an $AR(1)$ process with parameter 0.5; in period $F + 1$ they start believing the announced environmental plan.

Calibration

Parameter	Description	Value	Notes
β	Discount factor	0.9988	Real rate of 2% annually (NAWM-II)
φ	Inverse of Frisch elasticity	2	NAWM-II
ε	Elas. of subst. differentiated goods	3.8571	NAWM-II
α	Share of capital in production	0.2954	$\frac{i}{y} = 0.21$ (NAWM-II)
κ_P	Price adjustment costs	71.2043	NAWM-II
δ	Depreciation rate	2.5%	NAWM-II
θ	Growth rate of trend variables	1.0038	NAWM-II
κ_I	Investment adjustment cost	10.78	NAWM-II
π	SS inflation	1.005	ECB target
$\tilde{g}$	Public spending	0.4787	$g/y = 0.215$ (NAWM-II)
ϕ_π	Taylor rule coefficient	2.74	NAWM-II
ρ_r	Inertia of Taylor rule	0.93	NAWM-II
δ_x	Pollution depreciation	0.0035	Gibson and Heutel (2020)
$\tilde{e}^{row}$	Emissions in the rest of the world	14.8146	$\frac{e^{row}}{e} = 15.31$
χ	Convexity of abatement function	1.6	Gibson and Heutel (2020)
ν_M	Coefficient in the abatement function	0.1924	Gibson and Heutel (2020)
ν_E	Coefficient in the emission function	0.4390	$p^C = 65$ under $\mu = 1$

Table B.1: Calibrated parameters.

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